Geoid Models: History and application in GPS Surveying

Geoid Models

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Introduction

To help better understand this presentation I plan to touch on some history.

Before we delve into that let's review some terms

Datums

- NAD 27
- NAD 83
- NGVD 29
- NAVD 88
- MEAN SEA LEVEL
- ECEF earth centered earth fixed
- ITRS international Terrestrial Reference System by IERS international earth rotation and reference systems service

TERMS

SURFACES

- Sphere
- Oblate Spheroid is general shape of the earth
- Ellipsoid surface is created by rotating an ellipse around and axis
- Topographic surface: this is the surface we map
- Equipotential surface: based on gravity
- Geoid the equipotential surface of the earth

MODELS

- Geoid 90,93,96,99,2000,2003

2006, 2009 and others to follow

The above models represent an ever more precise “mapping” of the equipotential surface of the earth and because the surface of the earth is not static this “mapping” is a never ending task

Miscellaneous terms

- Grace: Gravity Recovery Climate experiment carried out by twin GRACE satellites
- Geoidal Undulation: Separation between the Geoids' surface and a Reference Ellipsoid
- Mean sea level is the average level of the ocean surface halfway between the highest and lowest levels recorded. We use mean sea level as a plane upon which we can reference or describe the heights of features on, above or below the ground.

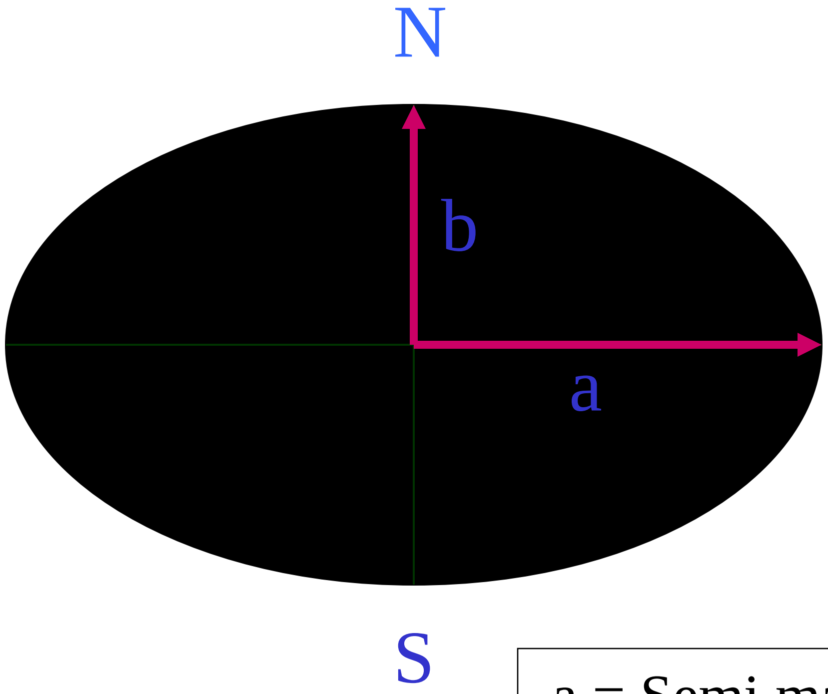
Wiser??

Ellipsoids or Spheroids

- The figure of the Earth

- Datums and Coordinate Systems
  - Horizontal and vertical

THE ELLIPSOID
MATHEMATICAL MODEL OF THE EARTH



a = Semi major axis

b = Semi minor axis

$f = \frac{a-b}{a}$ = Flattening

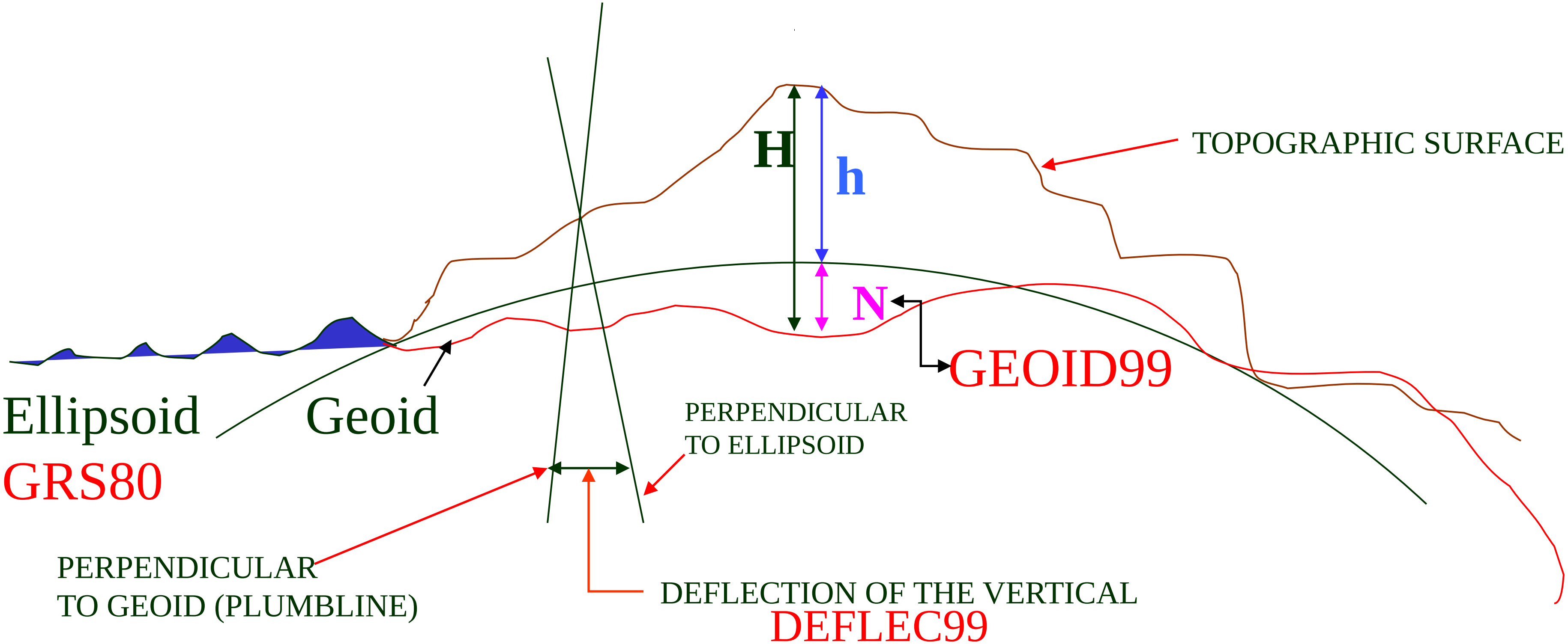
ELLIPSOID - GEOID RELATIONSHIP

H = Orthometric Height (NAVD 88)

h = Ellipsoidal Height (NAD 83)

N = Geoid Height (GEOID 99)

$H = h - N$



UNITED STATES ELLIPSOID DEFINITIONS

BESSEL 1841

a = 6,377,397.155 m 1/f = 299.1528128

CLARKE 1866

a = 6,378,206.4 m 1/f = 294.97869821

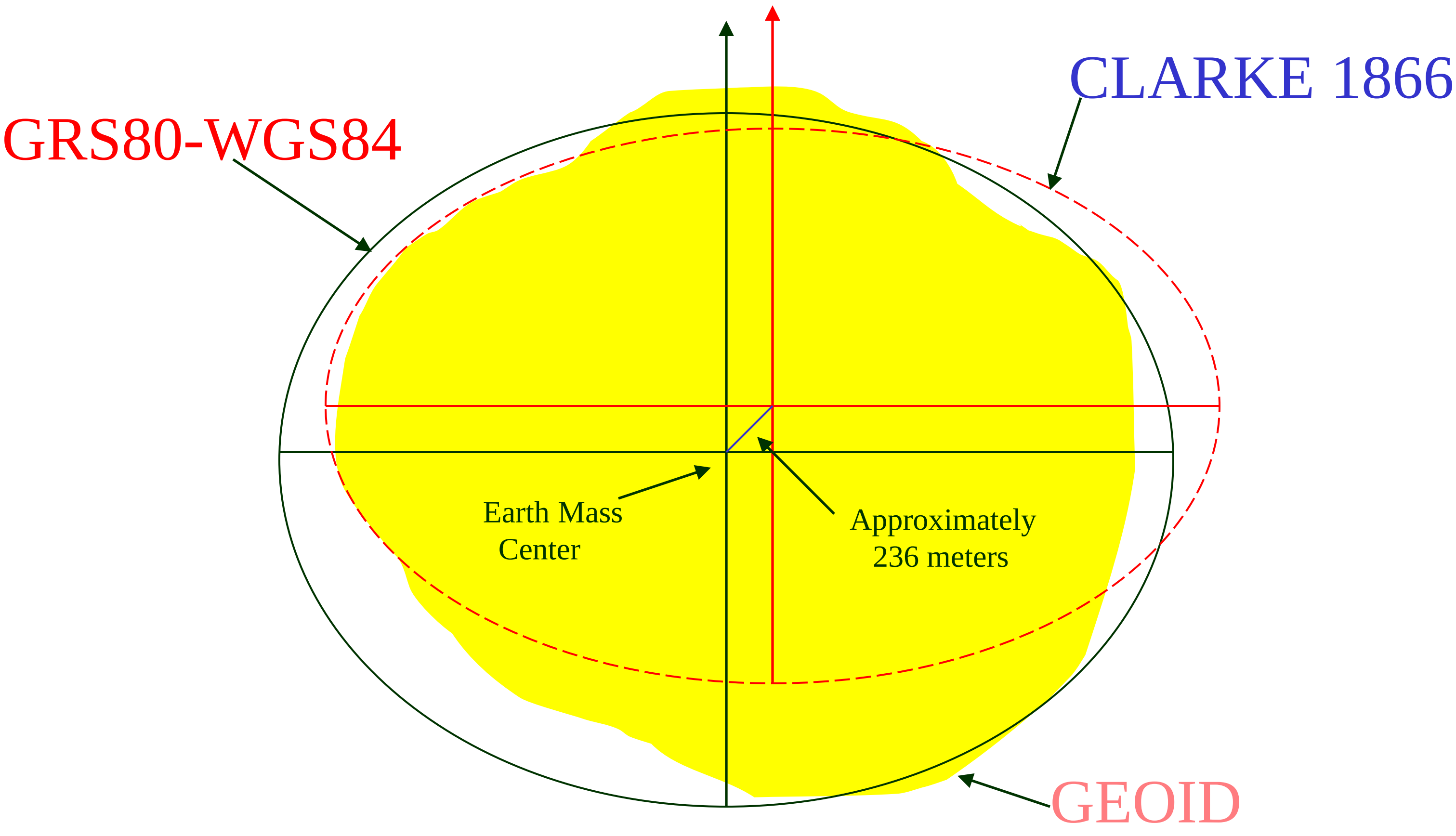
GEODETIC REFERENCE SYSTEM 1980 - (GRS 80)

a = 6,378,137 m 1/f = 298.257222101

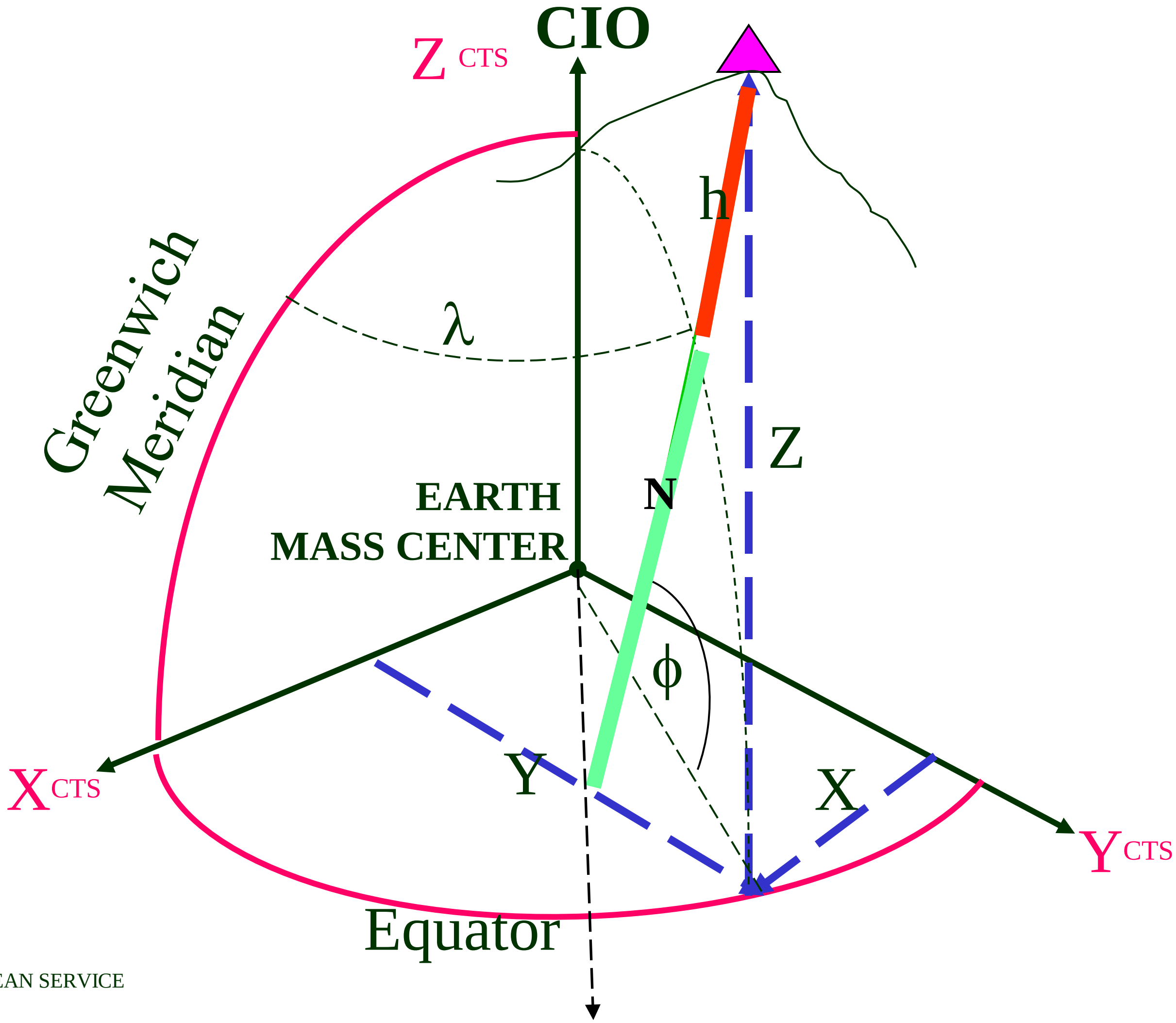
WORLD GEODETIC SYSTEM 1984 - (WGS 84)

a = 6,378,137 m 1/f = 298.257223563

THE GEOID AND TWO ELLIPSOIDS



CONVENTIONAL TERRESTRIAL SYSTEM (CTS)



CONVENTIONAL TERRESTRIAL SYSTEM (CTS)

TO TRANSFORM GEODETIC COORDINATES (ϕ, λ, h) TO RECTANGULAR COORDINATES (X, Y, Z)

$$X = (N + h) \cos\phi \cos\lambda$$

$$Y = (N + h) \cos\phi \sin\lambda$$

$$Z = [N + h (1 - e^2)] \sin\phi$$

Where $N = a / (1 - e^2 \sin^2\phi)$ — **Radius of Curvature in the Prime Vertical NOT the Geoid Height**

a = semi-major axis

$$e^2 = (a^2 - b^2) / a^2$$

From: Seeber "*Satellite Geodesy*"
Walter de Gruyter & Co. publishing, 1993

CONVENTIONAL TERRESTRIAL SYSTEM (CTS)

TO TRANSFORM RECTANGULAR COORDINATES (X, Y, Z)
TO GEODETIC COORDINATES (ϕ, λ, h)

$$Tan\phi = \frac{Z}{\sqrt{X^2+Y^2}}\left(1-e^2\frac{N}{N+h}\right)^{-1}$$

$$Tan\lambda = \frac{Y}{X}$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\phi} - N$$

From: Seeber "*Satellite Geodesy*"
Walter de Gruyter & Co. publishing, 1993

- Spherical earth models represent the shape of the earth with a sphere of a specified radius. Spherical earth models are often used for short range navigation (VOR-DME) and for global distance approximations. Spherical models fail to model the actual shape of the earth. The slight flattening of the earth at the poles results in about a twenty kilometer difference at the poles between an average spherical radius and the measured polar radius of the earth.

Ellipsoidal models define an ellipsoid with an equatorial radius and a polar radius. The best of these models can represent the shape of the earth over the smoothed, averaged sea-surface to within about one-hundred meters.

- Various units of length and angular distance have been used over history. The meter is related to both linear and angular distance, having been defined in the late 18th century as **one ten-millionth of the distance from the pole to the equator.**
- The most commonly used coordinate system today is the latitude, longitude, and height system.
- The Prime Meridian and the Equator are the reference planes used to define latitude and longitude.
- The geodetic latitude (there are many other defined latitudes) of a point is the angle from the equatorial plane to the vertical direction of a line normal to the reference ellipsoid.
- The geodetic longitude of a point is the angle between a reference plane and a plane passing through the point, both planes being perpendicular to the equatorial plane.
- **The geodetic height at a point is the distance from the reference ellipsoid to the point in a direction normal to the ellipsoid.**

VERTICAL DATUMS

MEAN SEA LEVEL DATUM OF 1929

NATIONAL GEODETIC VERTICAL DATUM OF 1929

(As of July 2, 1973)

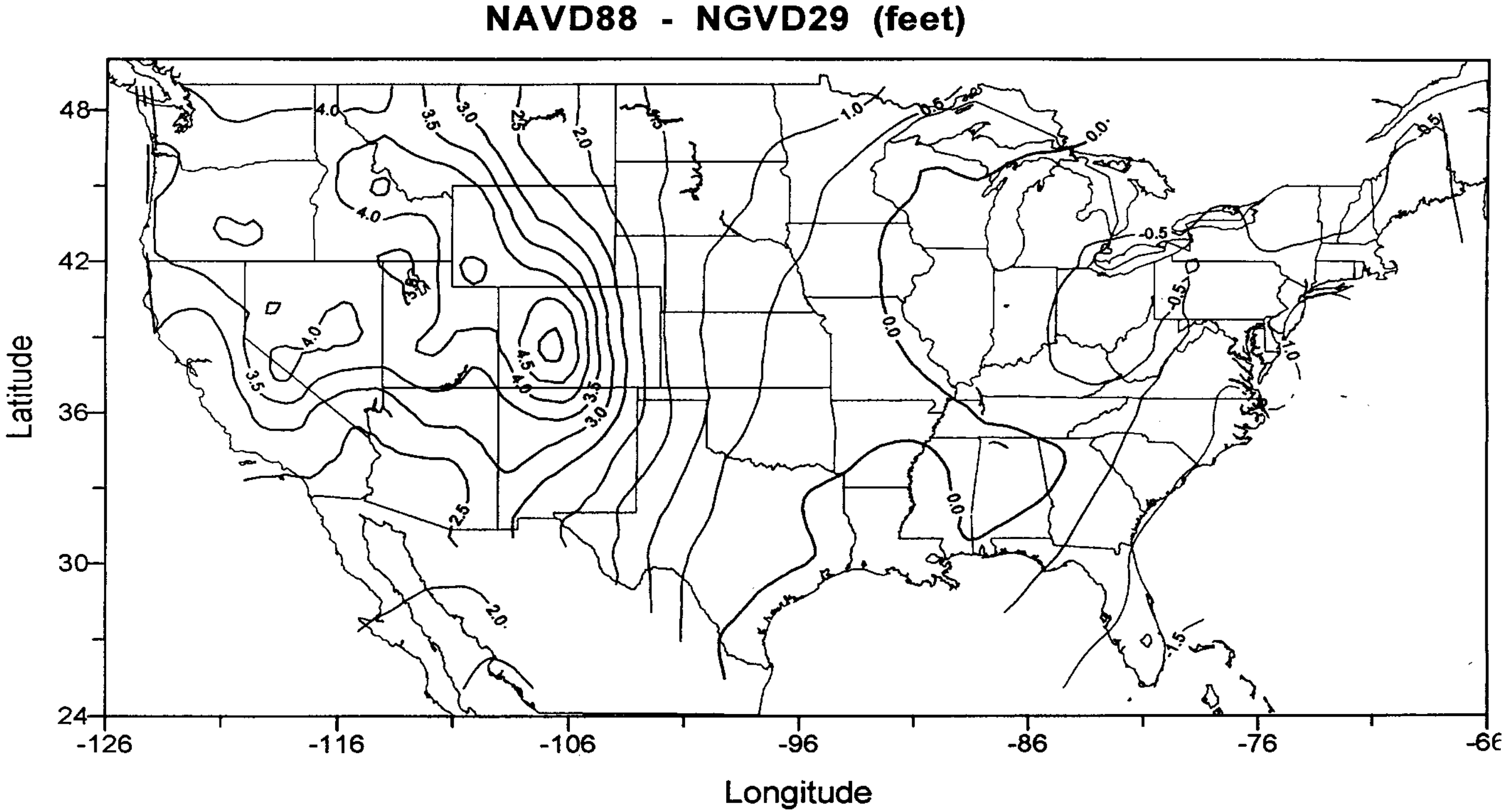
NORTH AMERICAN VERTICAL DATUM OF 1988

(As of June 24, 1993)

COMPARISON OF VERTICAL DATUM ELEMENTS

	NGVD 29	NAVD 88
DATUM DEFINITION	26 TIDE GAUGES IN THE U.S. & CANADA	FATHER'S POINT/RIMOUSKI QUEBEC, CANADA
BENCH MARKS	100,000	450,000
LEVELING (Km)	102,724	1,001,500
GEOID FITTING	Distorted to Fit MSL Gauges	Best Continental Model

NGVD 29 and NAVD 88



What is the GEOID

The Geoid is a mathematical model of the earth measured with gravity that corresponds with the mean ocean surface level on the earth- such as if the water were extended over the land. Because the surface is highly irregular however, there are different local geoids that are used to get the most accurate mathematical model possible for use in measuring vertical distances. By the National Geodetic Survey

There have been many definitions of the "geoid" over 150 years or so. Here is the one currently adopted at NGS:

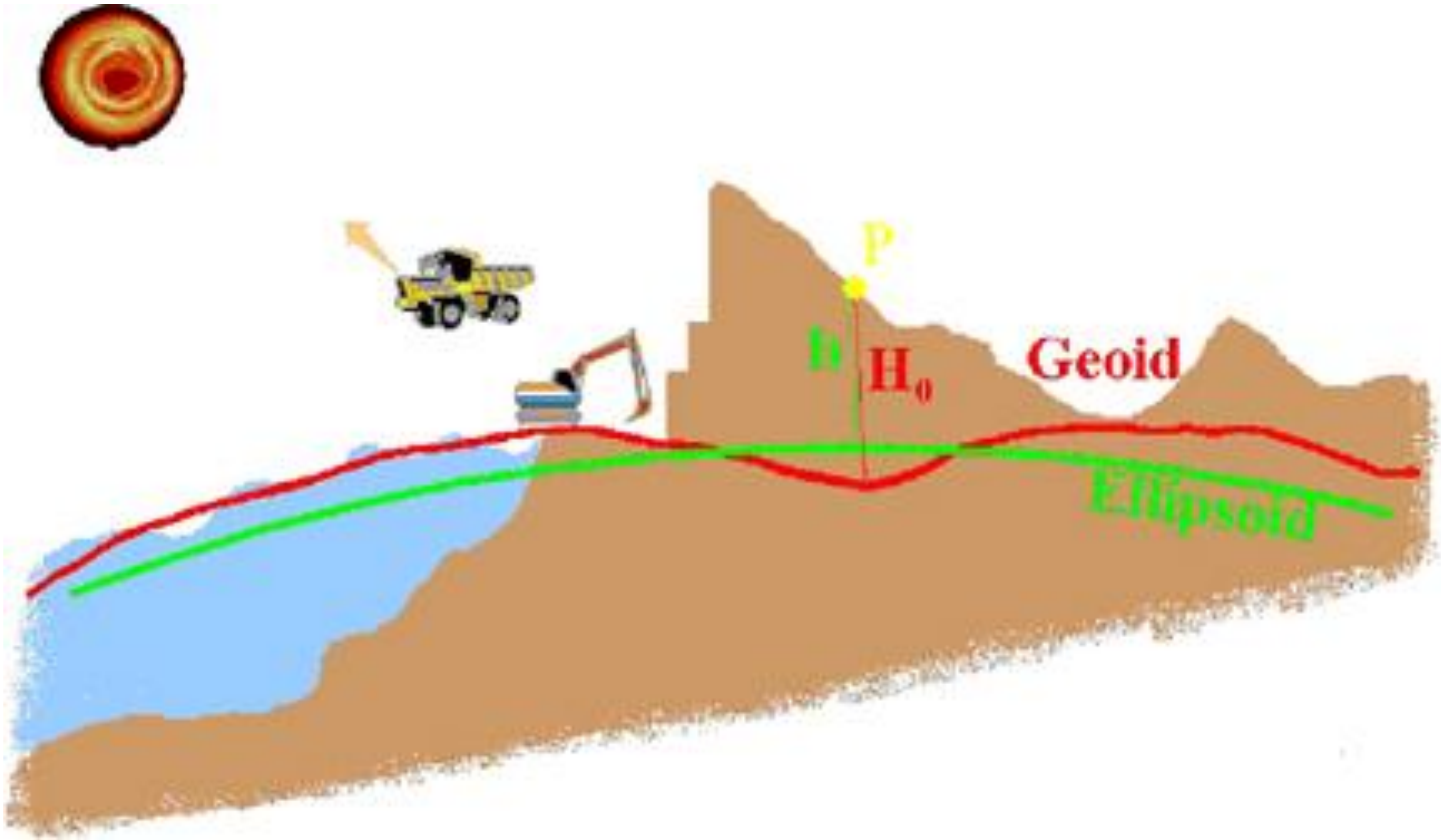
geoid: The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level

The Canadian Spatial Reference System describes this equipotential gravity surface, the geoid, very well:

"The geoid is theoretical only. You can't see it, touch it or even dig down to find it. Simply put, the geoid is the natural extension of the mean sea level surface under the landmass. We could illustrate this idea by digging an imaginary trench across the country linking the Atlantic and Pacific oceans. If we allowed the trench to fill with seawater, the surface of the water in the trench would represent the geoid. Not a bad way to imagine the geoid, but in reality not something we could easily do."

Even though we adopt a definition, that does not mean we are perfect in the realization of that definition. For example, altimetry is often used to define "mean sea level" in the oceans, but altimetry is not global (missing the near polar regions). As such, the fit between "global" mean sea level and the geoid is not entirely confirmable.

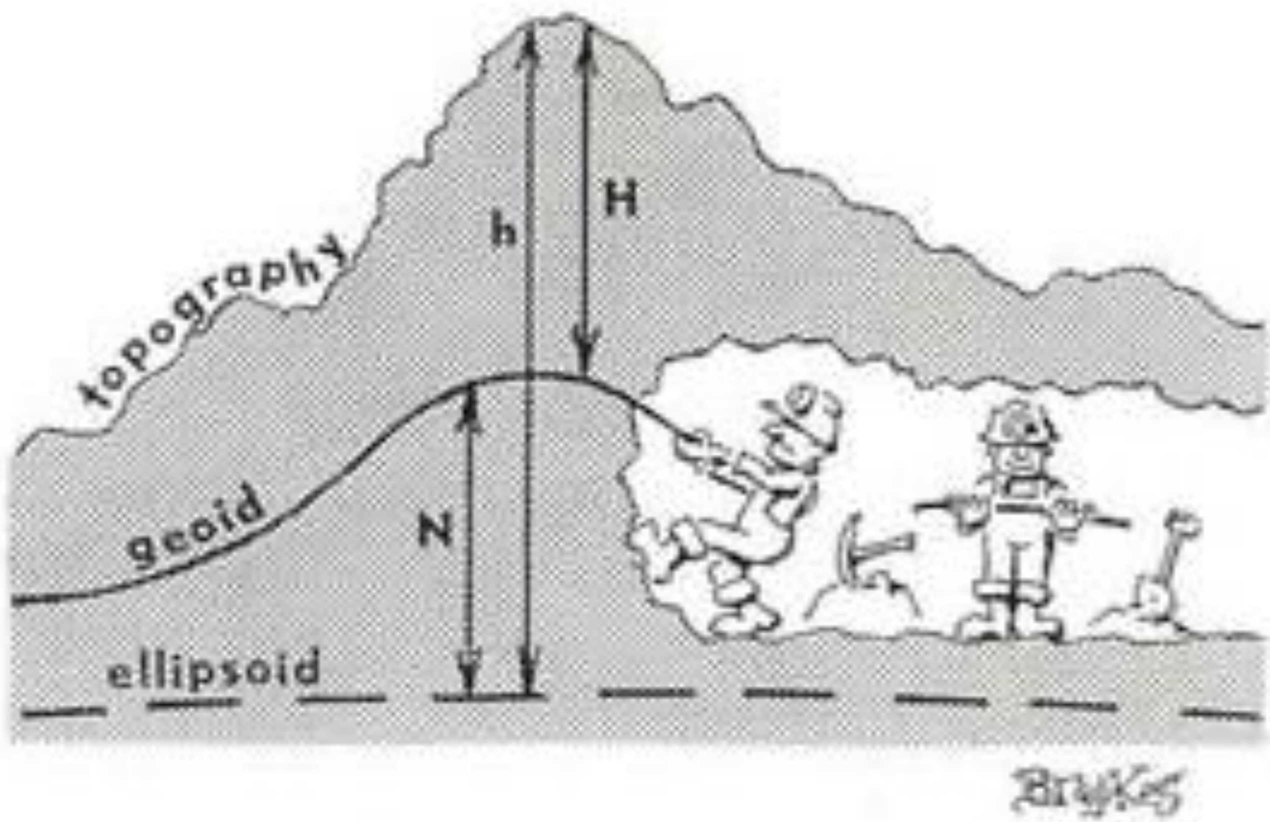
Also, there may be non-periodic changes in sea level (like a persistent rise in sea level, for example). If so, then "mean sea level" changes in time, and therefore the geoid should also change in time. These are just a few examples of the difficulty in defining "the geoid".



Digging an imaginary trench

The Geoid - How Do We Determine It?

As mentioned earlier, the geoid isn't something we can just go out and find. It is mean sea level plus the natural continuing of mean sea level under the landmass, our topography. This extension must be determined mathematically or modelled. The geoid model is actually based on gravity data collected worldwide. Once we determine the geoid, we can compute the difference between the two surfaces, the ellipsoid and the geoid anywhere in the country.



In Search of the Geoid

GEOID MODELS

U.S. NATIONAL MODEL -- GEOID99

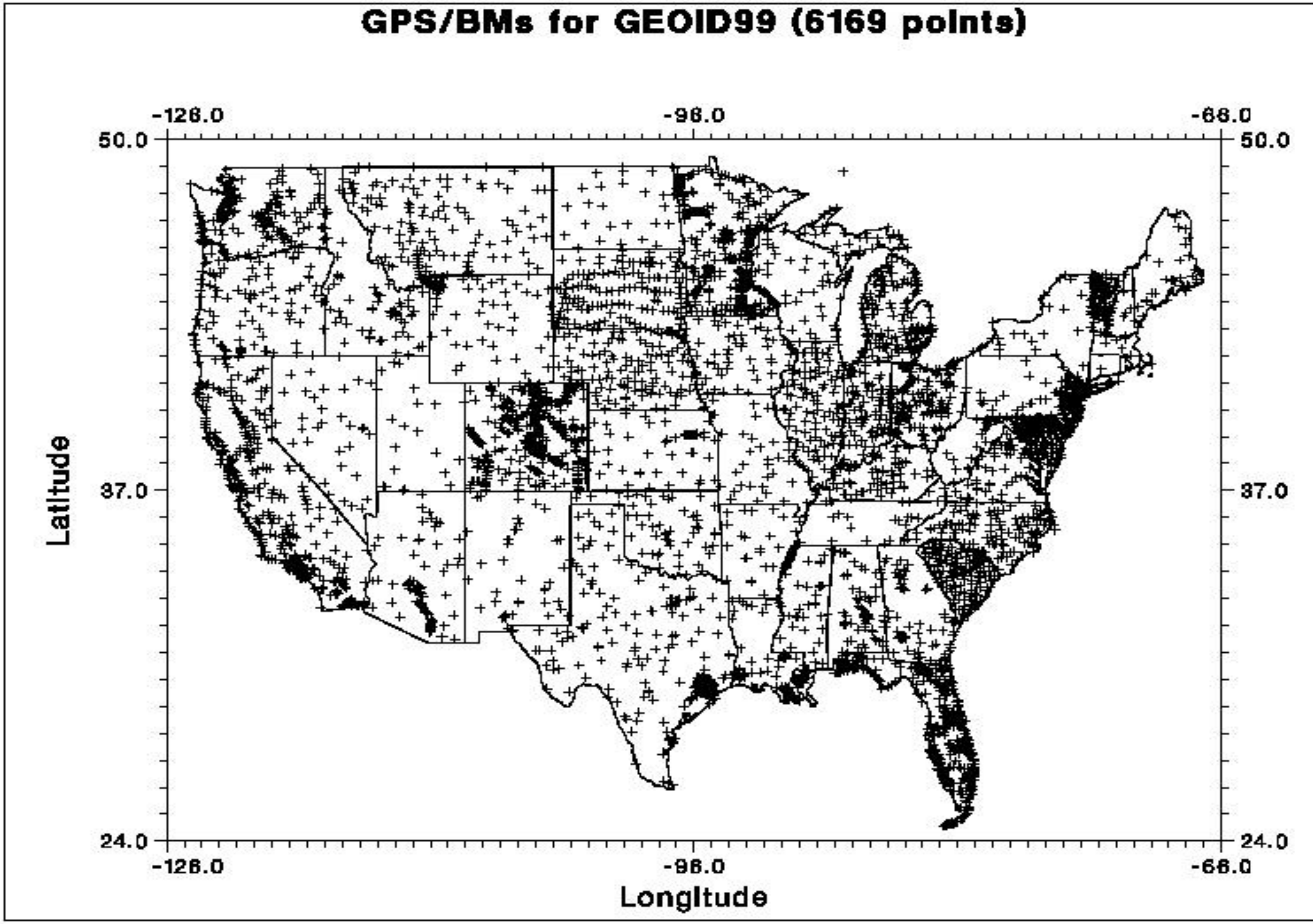
(http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl)

CANADIAN NATIONAL MODEL -- GSD95

http://www.geod.nrcan.gc.ca/products/html-public/GSDinfo/English/factsheets/gpsht_fact.html

GLOBAL MODEL -- EGM 96

(http://WWW.NIma.mil/GandG/wgs-84/egm96.html)


GPS/BMs for GEOID99 (6169 points)
-126.0
-96.0
-66.0
50.0
37.0
24.0
Latitude
Longitude

History of Geoid models

I started using GPS in 1986, and in 1990 I was surveying in the Grand Canyon. There were numerous challenges one of which it turned out to be is dealing with the effects of gravity differences. We ran levels to several Benchmarks and tested our results against the geodetic elevations, one benchmark was "out" by a meter and we assumed it was bad.

In those days just prior to Geoid 90 we would fix our orthometric elevations, obtained from Benchmark occupations and or leveling and “force fit” the GPS vectors. Soon after the project was completed we obtained Geoid 90 and discovered that the 1 meter “error” was actually due to the difference between the ellipsoid and the geoid.

There have been many subsequent refinements, such that we now feel confident that the model we use will give us a local Geoid elevation we can apply which will give us an orthometric elevation. Until Geoid 96 we had to be careful to create Psuedo ellipsoidal elevations on the stations we fixed Because the model was designed to be used with WGS 84 which differs at its center from GRS80 by about 2 meters

From 1996 forward the models that Surveyors use have been modified to avoid that problem so that we now can go directly from published ellipsoidal elevations to orthometric elevations. Presently most of us are just now switching to Geoid 2009.

below are some excerpts from Dennis Milbert's article " COMPUTING GPS-DERIVED ORTHOMETRIC HEIGHTS WITH THE GEOID90 HEIGHT MODEL"

Geoid heights relative to the GRS 80 Ellipsoid range from +75 to -104 meters worldwide. Within the conterminous United States , geoid heights will range from -5 to -50 meters. The negative values of geoid heights express the fact that the geoid is **Below the ellipsoid**

When one adjusts a set of GPS vectors, one must fix one or more points. Instead of fixing the ellipsoidal height of an existing GPS control point, fix the orthometric height of an existing benchmark, this has the effect of moving the three dimensional GPS network up or down to coincide with the local vertical datum, modified by the local geoid height model….

One should not rely upon one lone benchmark in gps height computation. One should connect into several benchmarks and check the misclosure at those marks. If an isolated error is identified, its source may be in the GPS, *or identification, or benchmark data (resets eg)*or the leveling but **NOT** in Geoid90.

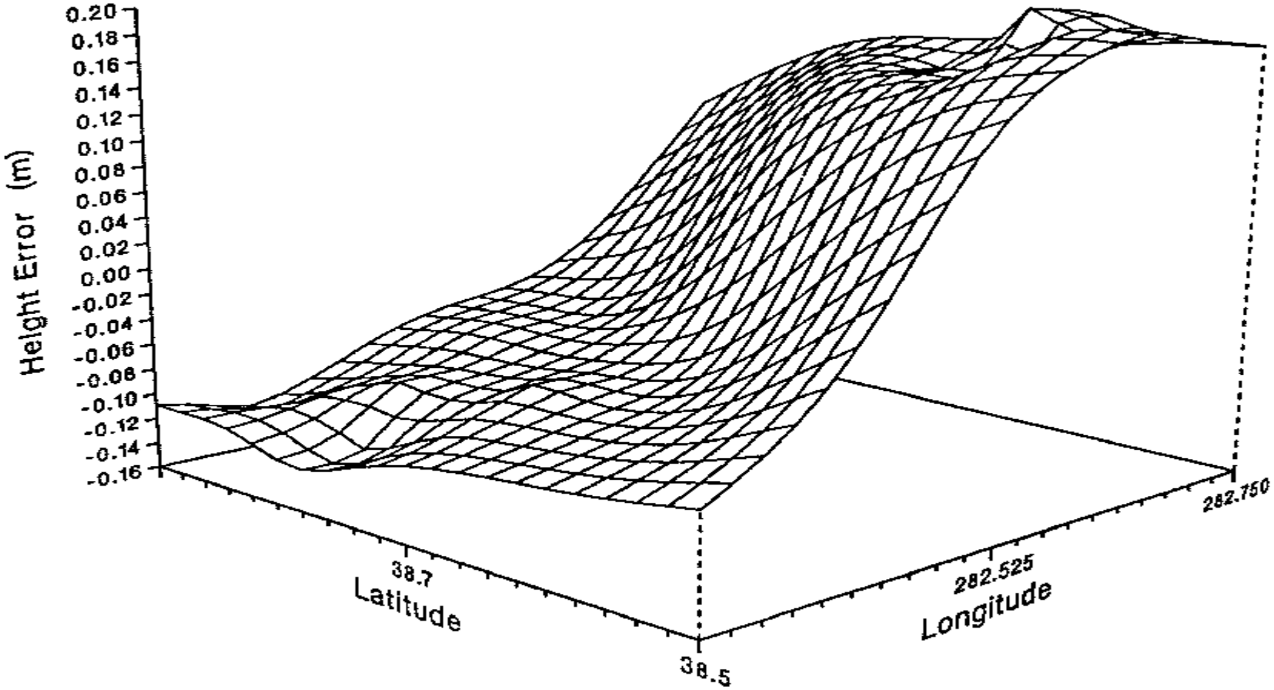
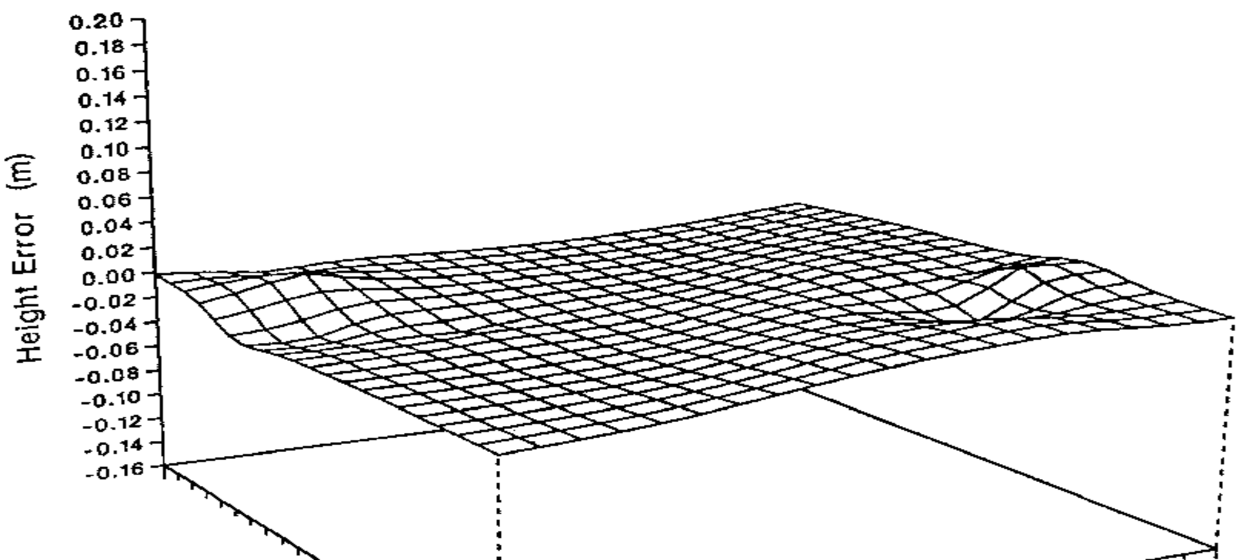
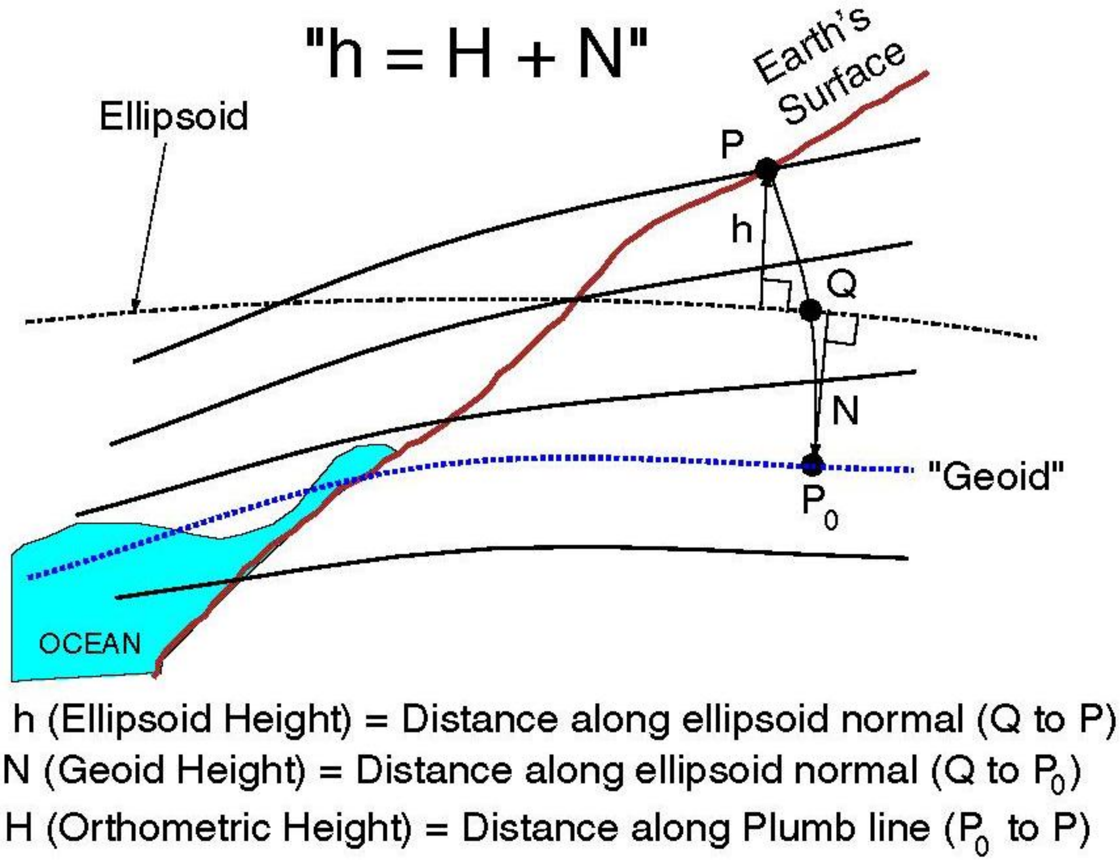


FIGURE 5a--Height Error using GPS, Leveling, and OSU89B Geoid Height





Note The plumb lines in this picture have been grossly exaggerated

GEODETIC DATA SHEET

$$H = h - N$$

$$1217.00 = 1117.09 - (-99.93)$$

$$1217.00 = 1217.02$$

```
   National Geodetic Survey,   Retrieval Date = AUGUST 16, 2001
HC0880 ***********************************************************************
HC0880  DESIGNATION -  EDGAR
HC0880  PID         -  HC0880
HC0880  STATE/COUNTY-  MO/PHELPS
HC0880  USGS QUAD   -  EDGAR SPRINGS (1954)
HC0880
HC0880                       *CURRENT SURVEY CONTROL
HC0880  ___________________________________________________________________
HC0880* NAD 83(1997)-  37 42 25.66456(N)    091 52 01.53003(W)     ADJUSTED
HC0880* NAVD 88     -       370.943  (meters)    1217.00    (feet)  ADJUSTED
HC0880  ___________________________________________________________________
HC0880  X           -    -164,620.928 (meters)                      COMP
HC0880  Y           -  -5,049,963.870 (meters)                      COMP
HC0880  Z           -   3,879,985.449 (meters)                      COMP
HC0880  LAPLACE CORR-           1.45  (seconds)                     DEFLEC99
HC0880  ELLIP HEIGHT-         340.49  (meters)                      GPS OBS
HC0880  GEOID HEIGHT-         -30.46  (meters)                      GEOID99
HC0880  DYNAMIC HT  -         370.656 (meters)    1216.06   (feet)  COMP
HC0880  MODELED GRAV-     979,846.5   (mgal)                        NAVD 88
HC0880
HC0880  HORZ ORDER  -  B
HC0880  VERT ORDER  -  SECOND     CLASS II
HC0880  ELLP ORDER  -  FOURTH     CLASS I
```

More Miscellaneous Information Just in Case You haven't had enough....

Two accuracy standards

(http://fgdc.er.usgs.gov/standards/status/swgstat.html)

local accuracy ------------- adjacent points

network accuracy ---------- relative to CORS

Numeric quantities, units in cm (or mm)

Both are relative accuracy measures

Do not use distance dependent expression

Horizontal accuracies are radius of 2-D 95% error circle

Ellipsoidal/Orthometric heights are 1-D (linear) 95% error

Relationships of meter to feet feet

- US Survey foot

1 meter = 39.37 inches exactly

1 meter = 3.2808333333333333etc feet

1 mile = 1.609 Kilometers approx

- International foot

1 inch= 0.0254 meters exactly

1 foot= 0.3048 meters exactly

1 meter = 3.280839895 feet

1 Us foot=1.000002 international foot

Or the US foot is the international foot Supersized

We're Living in the Future